How Prices Respond to Worked Orders

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Overview

Problem:

- In financial markets, it is standard practice to **work** large orders, i.e., to transact them piecemeal over a period of time (this is especially true in electronic markets).
- Despite the current prominence of worked orders in markets, little is understood theoretically about how worked orders should influence prices.

Analysis:

- We analyze how prices fluctuate in a market where: (1) all orders are worked, and (2) prices are set using a standard structural model of price formation.

Results

Results:

- Qualitatively our results match nicely with the following empirical findings:
    (1) Prices respond more to the early pieces of an order than the later pieces.
    (2) Prices revert, at least partially, after the order completes.
    (3) Slowly transacting the order results in lower transaction costs.

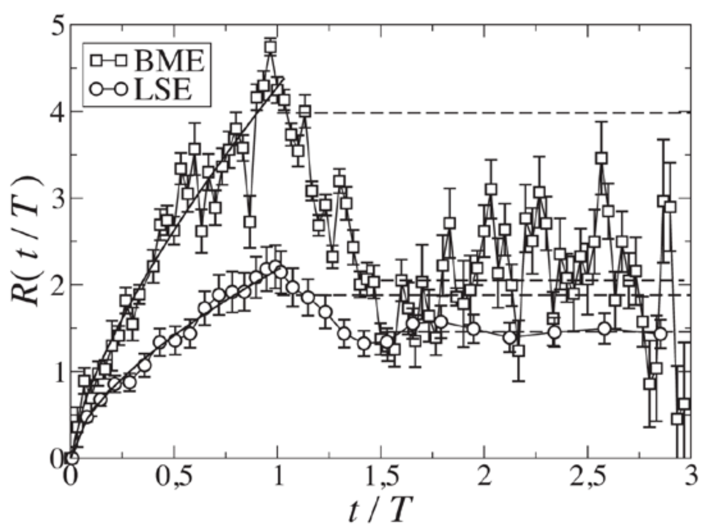


Figure 1: Plot of the price response to a worked order in the Bolsa de Madrid (BME) and the London Stock Exchange (LSE) (from Moro et al. (2009)). At time $t/T = 0$ the order starts and at time $t/T = 1$ the order completes. See Moro et al. (2009) for details.

Introduction

- Due to recent changes in the structure of financial markets (electronic automation), it has become relatively easy to work an order.

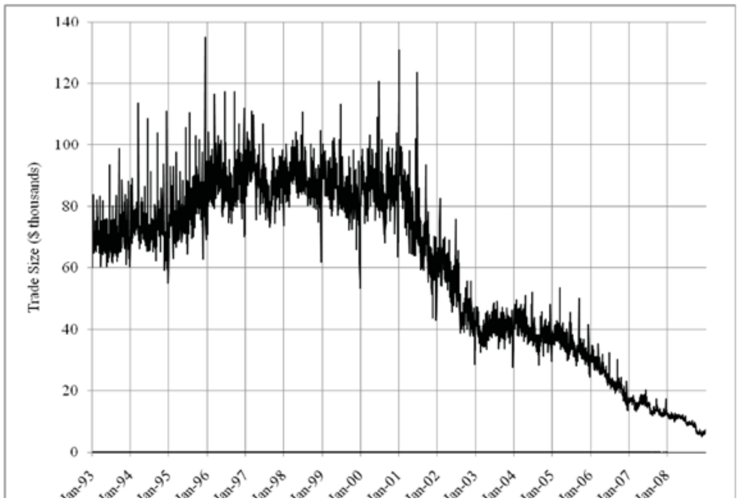
- As a result, many orders that previously would have transacted in one lot are now worked.

- This has had dramatic effects on order flow variables, with average transaction sizes plummeting and the number of transactions skyrocketing.

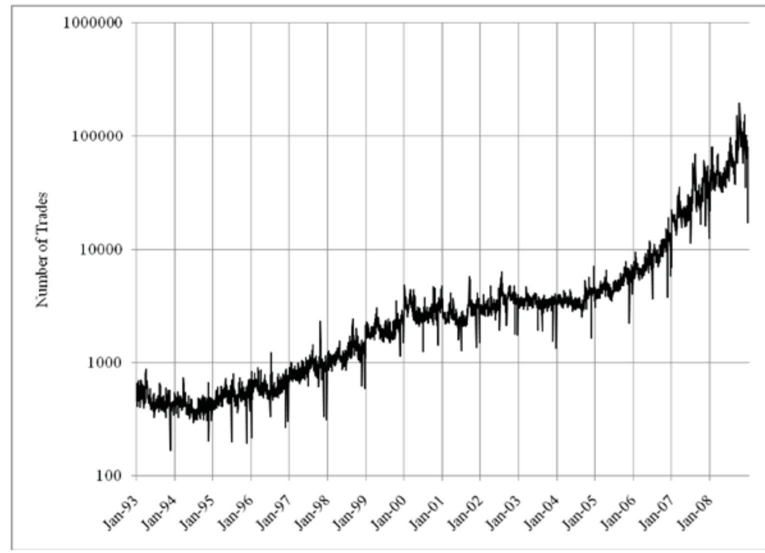
- There exists little previous work that analyzes how these changes affect price formation.

Changing Order Flow Variables

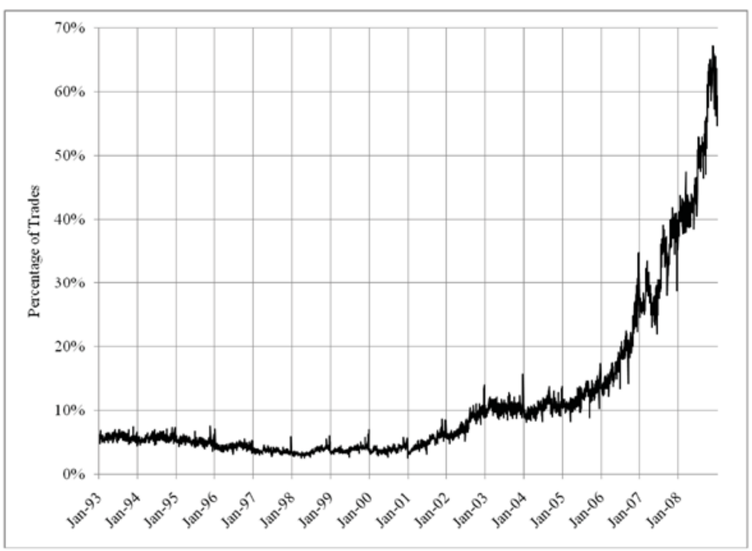
Panel A: Average Dollar Trade Size, 1993-2008



Panel B: Average Daily Number of Transactions, 1993-2008



Panel A: Percentage of Trades Less or Equal to $10,000



Figures from Chordia, Roll, and Subrahmanyam (2010).

Over the past 15 years, for NYSE listed stocks:

(1) Average transaction sizes have fallen dramatically.
(2) The number of transactions has increased 100 fold.
(3) The percentage of trades less than or equal to $10,000 has gone from ~6% to ~60% .

Changing Order Flow Variables

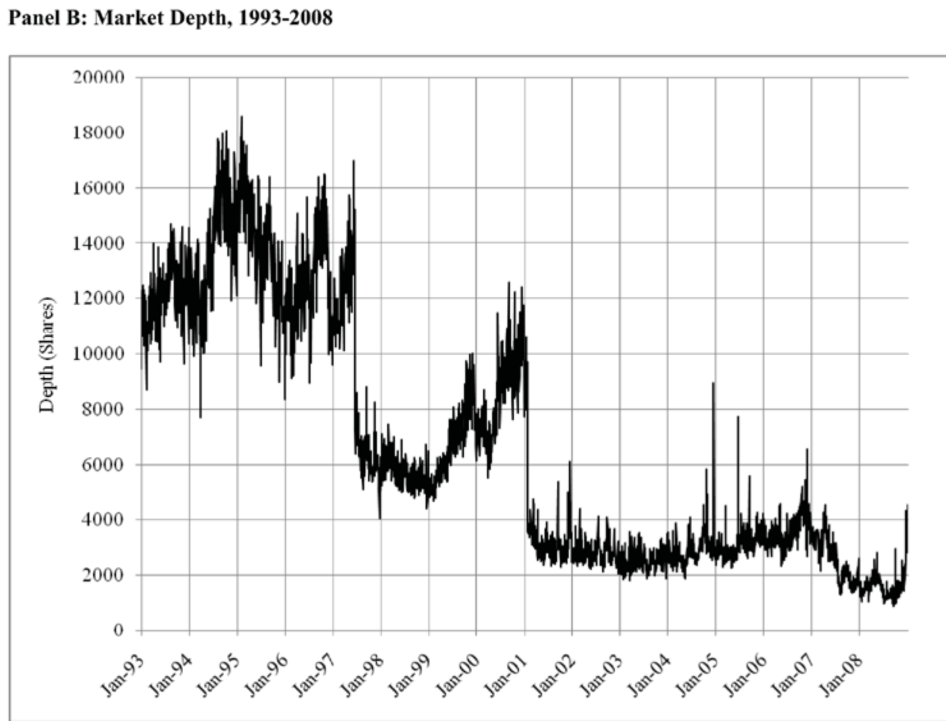


Figure from Chordia, Roll, and Subrahmanyam (2010).

- For NYSE listed stocks, the inside depth has decreased.
- Large orders are split not only because it is easy to do, but also because the lack of depth makes it necessary.

Price Response to Worked Orders

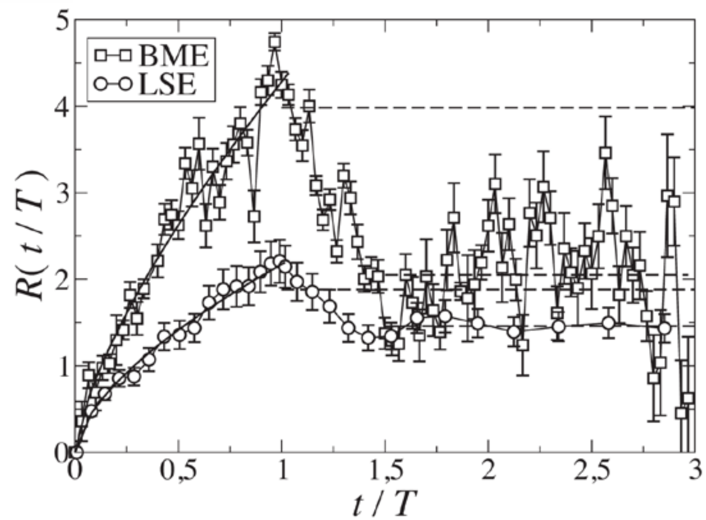


Figure 1: Plot of the price response to a worked order in the Bolsa de Madrid (BME) and the London Stock Exchange (LSE) (from Moro et al. (2009)). At time $t/T = 0$ the order starts and at time $t/T = 1$ the order completes. See Moro et al. (2009) for details.

Empirically, for worked orders:

(1) Prices respond more to the earlier pieces of the order than the later pieces so that price impact is a nonlinear and concave function of the total order size.

(2) Prices revert, at least partially, after the order completes.

(3) Slowly transacting the order results in lower transaction costs.

Literature

There is a disconnect with the empirical findings and the literature:

(1) It is often thought that price impact should be linear. (Kyle (1985) and Huberman and Stanzl (2004)).

(2) The price reversion after an order completes is thought to be due to compensation costs to liquidity providers or to market imperfections. (Stoll (2000)).

(3) There is little to no prior theoretical work that discusses speed of execution and price response.

Our Strategy

(1) We assume all orders are worked and model their execution in a simple way (all are VWAP orders).

(2) We assume transaction prices are set by liquidity providers using a standard structural model of price formation. We bracket the information available to liquidity providers between two extremes:

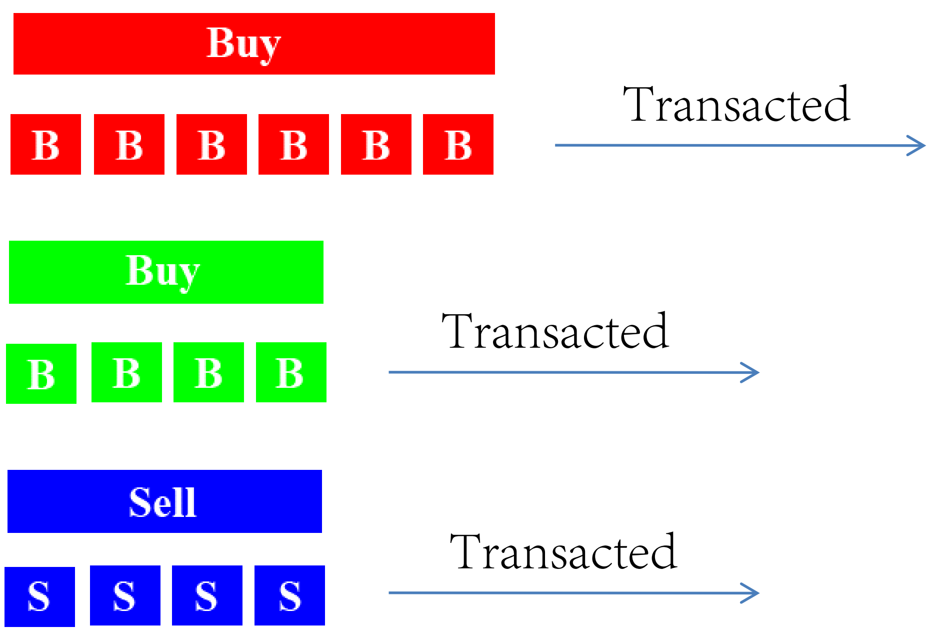
(a) Transactions are completely anonymous.

(b) Transactions are colored such that liquidity providers know how many pieces an order has transacted so far.

(3) We analyze how prices respond to worked orders under both scenarios.

Worked Orders

- Based on previous empirical results, we assume that orders are power law distributed with tail exponent $\alpha < 2$.
- We assume that all orders are worked, i.e., split into unit sized pieces and then transacted.



VWAP Execution

- We assume all orders are executed according to a VWAP strategy, i.e., they attempt to transact a constant fraction of volume in the market.
- An order is defined by three parameters:

$$\Psi_i = \{\varepsilon_i, \pi_i, N_i\}$$

ε	Sign of the order (whether it is a buy, +1, or a sell, -1).
π	Participation rate of the order, probability per unit time it transacts.
N	Size of the order in units of transactions.

Pricing Model

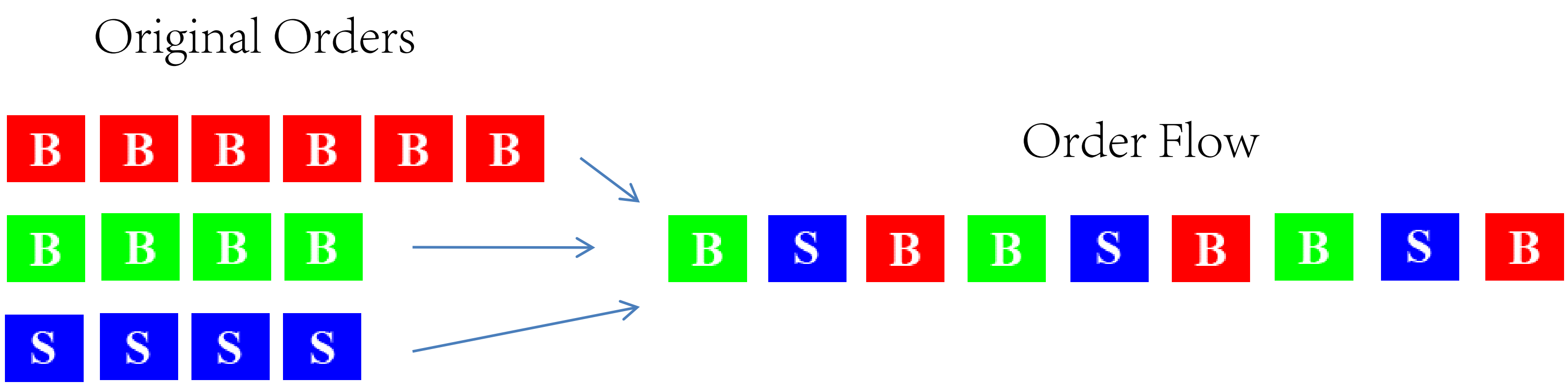
- All transactions are priced and cleared by liquidity providers according to the following equation:

$$p_t = p_{t-1} + (x_t - \hat{x}_t)\lambda + \epsilon_t.$$

p	Transaction price.
x	Sign of the transaction (whether it is a buy, +1, or a sell, -1).
$\hat{x}$	Predicted sign of the transaction based on the information available.
λ	Scale parameter that measures the degree of information asymmetry.
ϵ	Noise term.

Predictability of Order Flow

-Because orders are split over time, order flow is somewhat predictable.



Predictability of Order Flow

-The predictability of transaction sign depends on the information available to liquidity providers.

-We bracket this information between two extremes in our analysis:

(1) We assume that liquidity providers use a simple autoregressive model to predict transaction signs.

(2) We assume that liquidity providers can discern that an order is transacting and know its properties.

-In reality, we would expect markets to operate somewhere between these two extreme models.

Autoregressive Model

Order Flow

B S B B S B B S B

- First, we assume that liquidity providers cannot determine which transactions come from which orders, so that they use a simple autoregressive model to predict order flow.

$$\hat{x}_t = \sum_{k>0} a_k x_{t-k}.$$

a_k Autoregressive coefficient.

Colored Print Model

Order Flow

B S B B S B B S B

-Second, we assume that liquidity providers can discern each order separately, as if each transaction in the market had a color that can be used to associate it with its corresponding parent order.

$$\hat{x}_t = \sum_i \varepsilon_i \pi_i \mathcal{P}(n_i(t)),$$

$\mathcal{P}(n_i(t))$ Probability that the order is still transacting given that it has so far transacted *n* pieces at time *t*.

Price Impact Function

-To determine how prices respond to a worked order, we define a price impact function that measures the average price response through time for an order with a given set of parameters.

$$R(t|\Psi)$$

Propositions

PROPOSITION 1. *For the autoregressive model, the price response to an order while it is transacting is concave and is given by the following approximate equation:*

$$R\left(t_n|\Psi\right) \approx \frac{2\lambda}{\alpha}\pi^{\left(1-\frac{\alpha}{2}\right)}n^{\frac{\alpha}{2}}\left(1+\mathcal{O}\left[\frac{1}{n}\right]\right).$$

COROLLARY TO PROPOSITION 1. *For the autoregressive model, impact is larger for quickly transacted orders and smaller for slowly transacted orders.*

PROPOSITION 2. *For the colored print model, the price response to an order while it is transacting is concave and is given by the following approximate equation:*

$$R\left(t_n|\Psi\right) \approx \lambda[1+\alpha\log(n)]+\mathcal{O}\left[\frac{1}{n}\right].$$

Propositions

PROPOSITION 3. *For the autoregressive model, when an order completes, prices revert as an inverse power of the time since completion. The reversion is given by the following approximate equation:*

$$R(t_N+\tau|\Psi)\approx\lambda N\frac{1}{\tau^{\left(1-\frac{\alpha}{2}\right)}}\left(1-\mathcal{O}\left[\frac{1}{\tau}\right]\right).$$

PROPOSITION 4. *For the colored print model, when an order completes, prices revert exponentially in time to an amount* $\lambda\mathcal{P}(N)$ *less than the original impact. The reversion is given by the following approximate equation:*

$$R(t_N+\tau|\Psi)\approx\lambda\left\{1+\alpha\log(N)-\mathcal{P}(N)\left[1-(1-\pi)^{\tau}\right]\right\}+\mathcal{O}\left[\frac{1}{N}\right].$$

Interpretation

-When an order is split and transacted over time, it produces predictability in order flow that the market observes and uses to anticipate further transactions from the order.

-The later transactions of the order are more predictable than the earlier transactions, so they impact the price less.

-Prices revert after an order completes due to the market's anticipation of further transactions from the order that do not materialize.

-When liquidity providers cannot precisely determine which transactions come from which orders, then an order that is executed quickly looks like several orders transacting at the same time. Prices therefore react more to quickly executed orders.

Conclusions

-Due to recent changes in the structure of financial markets, it has become relatively easy to work an order.

-The working of orders has had dramatic effects on order flow variables, with average transaction sizes plummeting and the number of transactions skyrocketing.

-By applying a structural model of price formation to incrementally transacted orders, we analyze how prices fluctuate in a market where orders are worked.

-Our results reproduce several empirical findings that have otherwise been difficult to theoretically explain:

(1) Prices respond more to the early pieces of an order than the later pieces.
(2) Prices revert, at least partially, after the order completes.
(3) Slowly transacting the order results in lower transaction costs.

Thank You!